

Compute the following derivatives.

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx} \sqrt{x}(x^2 + 3x^4) = \frac{1}{2}x^{-1/2}(x^2 + 3x^4) + \sqrt{x}(2x + 12x^3)$$

$$\frac{d}{dx} \sqrt[3]{x}(x^6 + 2x) = \frac{1}{3}x^{-2/3}(x^6 + 2x) + \sqrt[3]{x}(6x^5 + 2)$$

$$\frac{d}{dx} \sqrt[4]{x}(\sqrt{x} + 5) = \frac{1}{4}x^{-3/4}(\sqrt{x} + 5) + \sqrt[4]{x}(\frac{1}{2}x^{-1/2})$$

$$\frac{d}{dx} (x^3 + \sqrt{x})(x^{1/3} + x^{3/4}) = (3x^2 + \frac{1}{2}x^{-1/2})(x^{1/3} + x^{3/4}) + (x^3 + \sqrt{x})(\frac{1}{3}x^{-2/3} + \frac{3}{4}x^{-1/4})$$

$$\frac{d}{dx} (x^{5/4} + 6)(x^2 + 6x + 2) = (\frac{5}{4}x^{1/4})(x^2 + 6x + 2) + (x^{5/4} + 6)(2x + 6)$$

$$\frac{d}{dx} (x^2 + 3x^{-1})(2x - 10x^2) = (2x - 3x^{-2})(2x - 10x^2) + (x^2 + 3x^{-1})(2 - 20x)$$

$$\frac{d}{dx} (4x^7 + 7x^2)(\sqrt{x} + x^{-3}) = (28x^6 + 14x)(\sqrt{x} + x^{-3}) + (4x^7 + 7x^2)(\frac{1}{2}x^{-1/2} - 3x^{-4})$$

$$\frac{d}{dx} (\frac{1}{x} + x^3)(x^{-1} + \frac{1}{x^3}) = (-\frac{1}{x^2} + 3x^2)(x^{-1} + \frac{1}{x^3}) + (\frac{1}{x} + x^3)(-x^{-2} - 3x^{-4})$$

$$\frac{d}{dx} (\frac{1}{x^2} + x^3)(4x - 90) = (-2x^{-3} + 3x^2)(4x - 90) + (\frac{1}{x^2} + x^3)(4)$$

Let $\frac{d}{dx} \sin(x) = \cos(x)$ and $\frac{d}{dx} \cos(x) = -\sin(x)$.

Compute:

$$\frac{d}{dx} \sin(x)(x^2 + \sqrt{x}) = \cos(x)(x^2 + \sqrt{x}) + \sin(x)(2x + \frac{1}{2}x^{-1/2})$$

$$\frac{d}{dx} \sin(x) \cdot \sin(x) = \cos(x) \sin(x) + \sin(x) \cos(x)$$

$$\frac{d}{dx} (\sqrt{x} + 5x^9) \cos x = (\frac{1}{2}x^{-1/2} + 45x^8) \cos(x) + (\sqrt{x} + 5x^9) (-\sin(x))$$

$$\frac{d}{dx} \cos(x)(3x^2 + 2x - 9) = -\sin(x)(3x^2 + 2x - 9) + \cos(x)(6x + 2)$$

$$\frac{d}{dx} \sin(x) \cos(x) = \cos x \cdot \cos x + \sin(x) \cdot (-\sin x) = \cos^2 x - \sin^2 x$$

$$\frac{d}{dx} \left(\frac{1}{x} + \frac{1}{\sqrt[3]{x}}\right) \sin(x) = \frac{d}{dx} (x^{-1} + x^{-1/3}) \sin x = (-x^{-2} - \frac{1}{3}x^{-4/3}) \sin x + \left(\frac{1}{x} + \frac{1}{\sqrt[3]{x}}\right) \cos x$$

$$\begin{aligned} \frac{d}{dx} \left(\frac{3}{x^2} + \frac{4}{\sqrt[3]{x^3}}\right) \cos(x) &= \frac{d}{dx} (3x^{-2} + 4x^{-3/2}) \cos(x) \\ &= (-6x^{-3} - \frac{12}{2}x^{-11/2}) \cos(x) + \left(\frac{3}{x^2} + \frac{4}{\sqrt[3]{x^3}}\right) (-\sin(x)) \end{aligned}$$

If $f'(2) = 3$ and $g'(2) = 4$, compute $(f \cdot g)'(2)$ given $f(2) = 10$ and $g(2) = 8$.

$$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$(f \cdot g)'(2) = f'(2) \cdot g(2) + f(2) \cdot g'(2)$$

$$= 3 \cdot 8 + 10 \cdot 4 = 64$$